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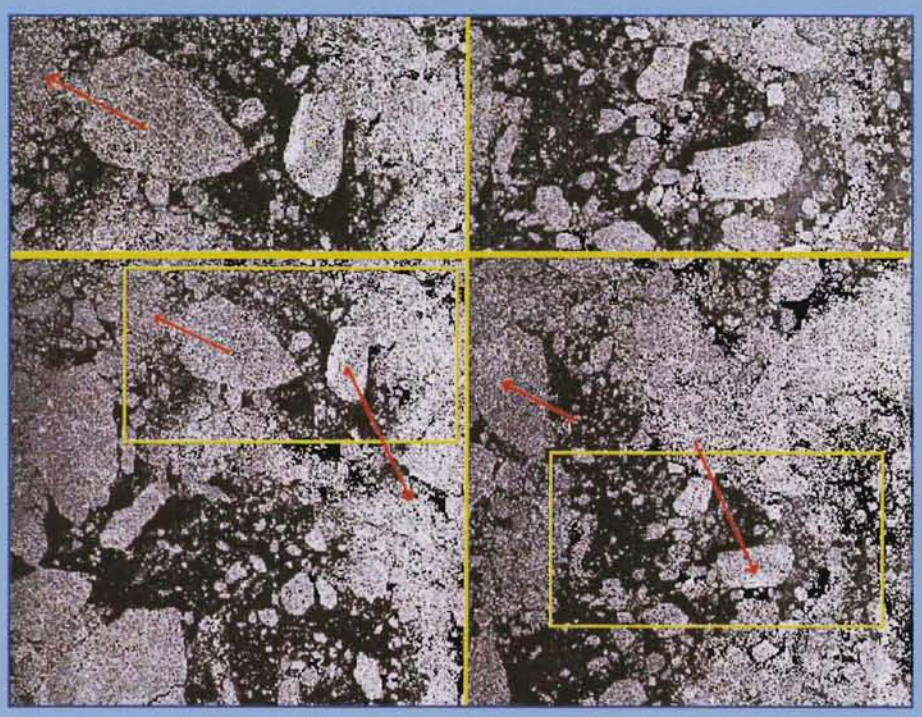
REINERT KORSNES

ICE TRACKING IN THE FRAM STRAIT FROM ERS-1 SAR IMAGES



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ICE TRACKING IN THE FRAM STRAIT FROM ERS-1 SAR IMAGES

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Preface

This report gives results from ice tracking in the Fram Strait September 1993 - September 1995 based on couples of ERS-1 SAR images approximately 3 days apart. The actual SAR images were low resolution data produced at the Tromsø Satellite Station (TSS). Chapter 2 describes the actual method for ice tracking and treatment of drift vectors. Chapter 2.2 gives a drift vector summary. Chapter 3 gives a treatment of data from the ERS-1 second ice period.

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Contents

Chapter 1

The Fram Strait ice flux

The major part (about 95 percent) of the ice that leaves the Arctic Ocean is conveyed with the Transpolar Ice Drift Stream which emerges through the Fram Strait (Vinje, T. and Finnekåsa, Ø., 1986). Consequently the ice flux through the Fram Strait reflects the net ice production in the Arctic Ocean and is of importance in connection with large scale circulation and heat budget studies of the Arctic (Untersteiner, 1988). Moreover, our measurements indicate a considerable temporal variation in the area ice flux through the strait. The area flux was in 1994 144,000 square km in January, as compared with 69,000 square km in February. This considerable variation indicates that there occurs a considerable variation in the rate of ice production and heat loss from the divergent areas upstream in the Arctic Ocean during the freezing season.

The area flux in March 1994 was about 134,000 square km. Upward Looking Sonars (ULS) moored about 50 m below the sea surface will provide the ice thickness distribution estimates for these periods. This will give more precise information about the ice flux from the Arctic Ocean.

This report particularly gives results from automatic ice tracking in the Fram Strait using time sequences of ERS-1/SAR images from the ERS-1 Phase D (second ice period January-March 1994). A similar work was made for the ERS-1 first ice period (January-March 1992) (Korsnes, 1994a). These results are preliminary since the data coverage is very limited and do not utilize all relevant data stored in the ESA archives from the ERS-1 ice periods.

Due to the increased ice field deformation and dynamics in the Fram Strait (compared to most of the Arctic Ocean), ice tracking is normally more difficult in this area. Hence this study utilizes a method with a 2100 m by 2100 m 'test window' for cross correlation (note that the high resolution data could enable an even smaller test window). However, this study concerns data from automatic ice tracking only from the winter season. The rest of the drift data is from visual image inspection. A general and robust code for ice automatic drift monitoring via direct observations in the Fram Strait still may be improved.

Buoys dropped in the Arctic Ocean tend to be on old and thick ice when they reach the Fram Strait. We may believe thick ice in periods of low ice concentration tend to drift somewhat differently compared to the average ice field. Sets of buy drift estimates may therefore not directly represent the local long term time average of the ice drift ve-

locity in the Fram Strait. Also, sets of buoy drift estimates may tend to under-represent periods with low concentrations of old ice in the Fram Strait. Situations with prevailing storms from North, for example, will give divergence and production of new ice upstream the Fram Strait. This mechanism contributes to higher average velocity of new ice compared to old ice. A possible statistical relation between ice thickness/concentration and drift velocity may therefore give bias in estimates of the long term ice drift through the Fram Strait based on drift buoys. Effective ice tracking may contribute to reduce the uncertainty within these approaches and it is of general interest in studies on controlling mechanisms for ice drift and indicators for climate variations.

Chapter 2

Data processing

2.1 Ice tracking method

The ice tracking within this work consists of a manual and an automatic operation. The manual operation is to identify several identical undeformed areas in subsequent ERS-1 SAR images giving a set of normally 3-6 drift vectors. The automatic part of the procedure is automaticly to give a dense sampling of the ice drift between the image times within the actual area. Chapter 3 gives results from such data treatment for the period January - March 1994.

2.2 Ice drift summary

This chapter gives a summary of the estimates of ice drift within the present data coverage of ERS-1 SAR images approximately 3 days apart. In order to give a uniform procedure for all the data, the ice drift vectors are from visual inspection and by directing the cursor on a computer screen showing plots of the data (2-6 vectors from each image pair).

Table 2.1: Monthly vector average of ice drift in the Fram Strait for ice "particles" starting 5 - 8 deg West and 78.5 - 79.5 deg North.

Month (1993)	# image pairs	Velocity (m/s)	Direction (deg)
06	3	.018	134.
07	6	.053	282.
08	2	.092	241.
09	5	.081	288.
10	26	.113	207.
11	3	.034	122.

The ice drift summary is condition on drift vectors starting between 5 and 8 degree West and between 78.5 and 79.5 degree North. The data for the statistics i Tables 2.1 - 2.3 are average vectors from each image pair from where there was possible to extract ice drift vectors. This means that the vectors from each image pair in total contribute with weight 1 in the total statics. The rationale for this approach is to decrease the bias in the statics due to difficult ice tracking from data from periods where storms/swell (from east) break the ice. During periods with cold winds from North and East the ice tracking is presumably simplest. Hence one may risk to get

a correlation between missing data and low ice drift (wind from South and East).

Figure 2.1 illustrates that neighboring ice floes in relatively warm periods (low ice concentration) may have different ice drift. This example is from 30 July - 2 August 1994 (compare this to Figure 3.2). The size of an ice floes may affect its drift speed. This may lead to bias in the ice drift estimates from ice tracking using satellite images. It is easier to identify large ice floes than small ice floes in different satellite images. Large ice floes may drift somewhat different compared to small ice floes. Extra thick ice may drift differently compared to the main ice field. This may give bias in ice drift observations.

Table 2.2: Monthly vector average of ice drift in the Fram Strait for ice "particles" starting 5 - 8 deg West and 78.5 - 79.5 deg North.

Month (1994)	# image pairs	Velocity (m/s)	Direction (deg)
01	21	.281	213.
02	11	.165	221.
03	11	.257	218.
07	1	.040	230.
08	9	.040	230.
09	12	.112	204.
10	8	.219	211.
11	16	.129	217.
12	10	.272	210.

The individual ice drift vector data can be obtained via a request to the author. Figure 2.2 gives an example of the format for these (ASCII) data.

Table 2.3: Monthly vector average of ice drift in the Fram Strait for ice "particles" starting 5 - 8 deg West and 78.5 - 79.5 deg North.

Month (1995)	# image pairs	Velocity (m/s)	Direction (deg)
02	25	.310	214.
03	15	.361	214.
04	6	.264	206.
05	16	.128	190.
06	4	.107	193.

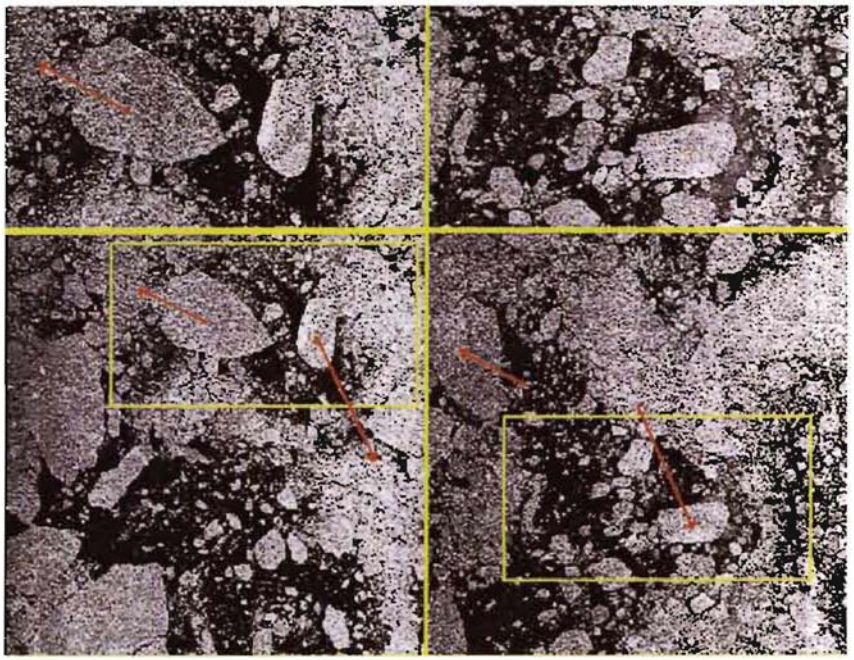

Figure 2.1: Example of variable ice drift in the study area.

```
Drift period : 1993.10.31 20:48:19 -- 1993.11.02 13:07:58
      lat,     long     -->   lat,     long
      79.128    -8.822 -->  78.992    -8.962
      79.181    -8.461 -->  79.048    -8.573
      79.232    -6.910 -->  79.074    -6.919
      79.156    -7.388 -->  79.008    -7.448
      79.116    -6.703 -->  78.961    -6.823
      79.026    -7.232 -->  78.877    -7.398
      78.974    -6.064 -->  78.808    -6.272
```

Figure 2.2: Example of ice drift data file format.

Chapter 3

Ice drift during ERS-1 second ice phase

3.1 Ice tracking during ERS-1 second ice phase

This section gives estimates of ice drift through the Fram Strait during the ERS-1 Phase D. The experimental ice tracker developed at the Norsk Polarinstitutt (Korsnes, 1994b) provides sampling of the ice displacement each 3 day relating pairs of ERS-1/SAR images. Figure 3.1 illustrates the actual observation window.

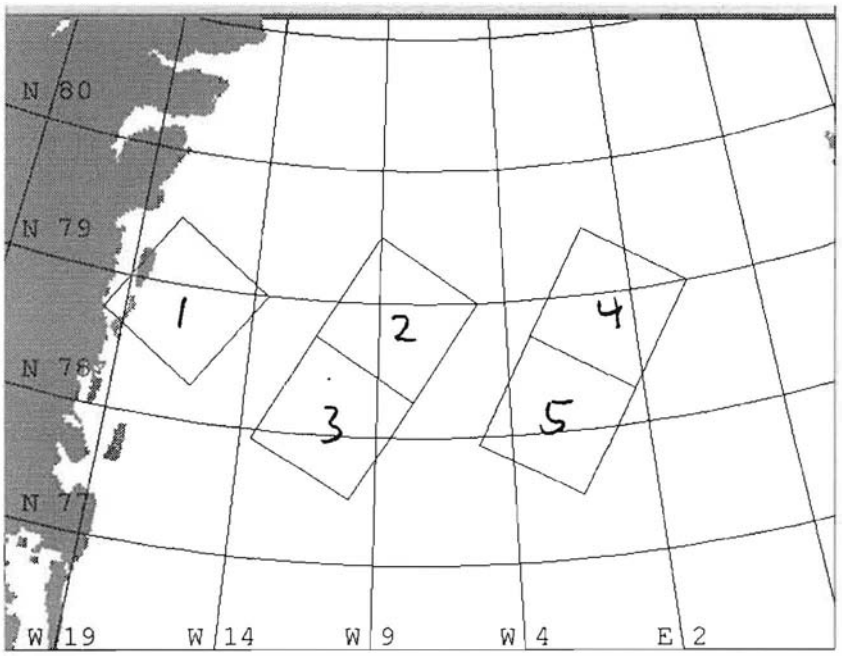

Figure 3.1: The present study ERS-1/SAR scene coverage during the ERS-1 Phase D.

The Scene #4 and #5 (cf Figure 3.1) almost hit the Eastern part of the ice extent

during the whole observation period. And the ice covered by Scene #1 was almost stagnant. Hence only the ice covered by Scene #2 and #3 were subject to ice tracking.

The ice tracker defines a 1 km spaced grid over this region and tries to find the ice displacement from each grid point for each image time (except the last) for the subsequent 'target' image time 3 days later. It eliminates 'flyers' (point miss-match) using a 'rigid area filter' passing through only points with high scores for matching or included in clusters of minimum 3 points exhibiting rigid object motion (rotation/shift). Figures 3.2 and 3.3 show examples where ice tracking in the Fran Strait respectively seems to be relatively simple (16-19 January 1994) and difficult (15-18 February 1994).

3.2 Discussion

Tables 3.1 - 3.9 show the summarized drift estimates in the Fram Strait starting within a circle with radius 25 km and with centers at latitudes N $79.3°, 79.0°, 78.7°$ and longitudes W $11°, 10°, \ldots, 6°$. Appendix A explains the symbols in these tables. The tables show slight easterly components in the gradient of the ice velocity between W 11° and W 6° in the Fram Strait. Since the ice covered by Scene #5 is always almost stagnant, we assume a distinct shear zone close to W 11°. The drift direction is normally in the range $190° - 220°$ (parallel with the continental shelf slope). The rotation seems to be positively correlated with drift velocity.

Ice particles starting at N 79° W 7° drifted south in three days with an average speed of 0.27 m/s in January. The figures for February and March are respectively 0.13 m/s and 0.25 m/s. The width of the ice drift stream starting from N 79° W 13° is about 200 km. The above ice drift figures should be roughly representative for the whole ice stream. This gives an area flux of about 144,000 square km in January (assuming 100 percent ice concentration). The values for February and March are about 69,000 and 134,000 square km. The ERS-1 ice phases (3 day repeat coverage) could in principle provide a complete time series of ice drift in the Fram Strait. Then there would be no uncertainty in the estimates due to sparse sampling in time. Gaps in the data series given by Tables 3.1 - 3.6 are caused by interruptions of the operation of the Tromsø Satellite Station (TSS) and the ERS-1 satellite. If the gaps were due to ice tracking problems caused by, for example, storms from South-East, one would introduce bias in the ice drift observation frequency. Missing data in Tables 3.4 - 3.6 (ie data from February) for some positions along the East-West observation profile are due to ice tracking problems caused by strong wind from South and East. In this case the observation nearest to the position N 79° W 7° replace the missing drift estimate in this simple calculation above.

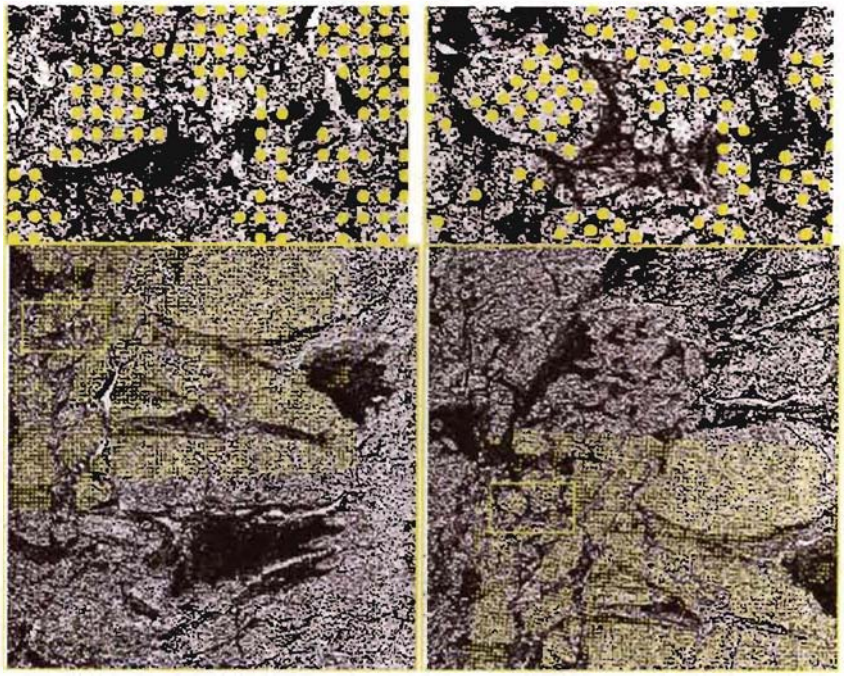

Figure 3.2: Example where it is simple to track ice through ERS-1/SAR scenes. The data are form 16 (left) - 19 (right) January 1994 - Scene #2 on Figure 3.1. The upper images represent the small framed area given below. The dots illustrate positions for successful ice displacement sampling. ©ESA (1994).

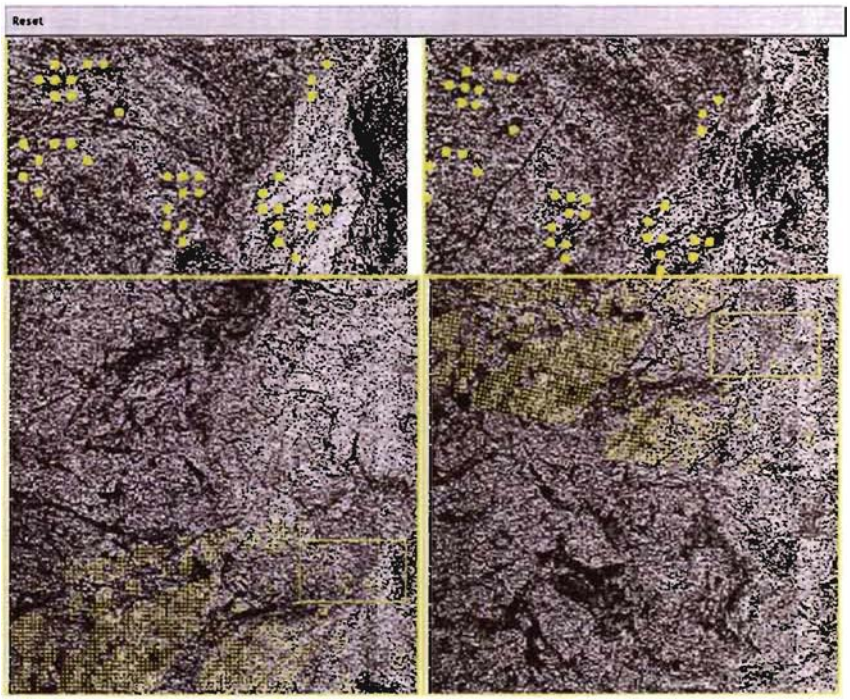

Figure 3.3: Example where it is difficult to track ice through ERS-1/SAR scenes. The data are 15 (left) - 18 (right) February 1994 - respectively Scene #2 and #3 on Figure 3.1. See caption of Figure 3.2 above. ©ESA (1994).

Table 3.1: Ice drift Fram Strait January 1994 starting in circles of radius 25 km and centers at N 79.3°.

Start		3 days mean		Deformation					Rota-	Prediction	#
date	longi-	Vel	Dir					θ_1	tion	error	data
(1994)	tude	(m/s)	(deg)	v_1	v_2	$v_1 v_2$	v_1/v_2	(deg)	(deg)	(m)	points
01.01	-9.	0.32	199	1.35	0.87	1.17	1.56	136	16.6	1919.3	547
	-8.	0.33	199	1.20	0.88	1.07	1.36	136	12.7	1815.5	888
	-7.	0.36	198	1.24	0.81	1.01	1.53	127	13.9	1792.9	686
	-6.	0.40	197	1.26	0.85	1.06	1.48	131	12.0	1776.5	447
01.04	-9.	0.30	206	1.04	0.97	1.02	1.07	124	9.9	244.6	331
	-8.	0.31	205	1.03	0.99	1.02	1.05	122	9.6	356.9	532
	-7.	0.32	204	1.00	0.99	0.99	1.01	99	9.3	154.3	651
	-6.	0.33	204	1.03	0.97	1.00	1.06	123	9.9	529.3	717
01.07	-9.	0.25	200	1.07	0.96	1.03	1.11	105	-0.2	354.7	684
	-8.	0.25	200	1.09	0.93	1.01	1.17	106	0.0	623.5	1051
	-7.	0.25	199	1.10	0.88	0.97	1.24	105	0.8	677.8	765
	-6.	0.25	199	1.08	0.92	0.99	1.17	104	-1.0	957.0	561
01.10	-9.	0.19	194	1.13	0.92	1.04	1.24	121	11.2	499.3	524
	-8.	0.20	193	1.07	0.95	1.02	1.13	121	8.8	845.6	938
	-7.	0.21	192	1.09	0.93	1.02	1.17	117	7.1	940.5	861
	-6.	0.22	192	1.08	0.93	1.01	1.16	123	3.9	1148.6	595
01.13	-9.	0.34	196	1.22	0.79	0.97	1.54	131	20.7	3135.5	402
	-8.	0.35	196	1.17	0.93	1.10	1.26	104	10.2	1023.0	516
	-7.	0.36	195	1.13	0.92	1.04	1.23	109	7.7	553.9	268
	-6.	0.37	195	1.09	0.91	0.99	1.20	114	3.4	472.0	37
01.16	-9.	0.17	196	1.14	0.87	1.00	1.31	133	9.4	1177.1	566
	-8.	0.18	195	1.07	0.95	1.02	1.12	127	5.4	1076.6	927
	-7.	0.19	196	1.02	0.99	1.01	1.03	133	3.1	262.8	828
	-6.	0.19	196	1.00	1.00	1.00	1.00	159	2.5	81.8	397
01.19	-9.	0.26	218	0.99	0.90	0.89	1.11	32	0.7	524.7	348
	-8.	0.27	219	0.99	0.93	0.92	1.07	34	3.0	501.0	614
	-7.	0.27	220	0.98	0.95	0.93	1.03	47	4.6	314.3	565
	-6.	0.28	221	0.99	0.97	0.96	1.02	67	4.8	151.6	211
01.22	-9.	0.38	206	1.26	0.72	0.91	1.75	130	11.5	1245.0	236
	-8.	0.41	205	1.25	0.83	1.04	1.49	129	10.9	1052.3	391
	-7.	0.42	205	1.24	0.83	1.02	1.49	133	11.2	1156.0	276
	-6.	0.44	205	1.25	0.87	1.08	1.45	155	14.0	740.0	50
01.31	-9.	0.09	213	1.01	0.96	0.97	1.06	145	2.9	431.5	773
	-8.	0.10	213	1.01	0.93	0.94	1.09	146	3.1	463.3	889
	-7.	0.10	213	1.02	0.89	0.91	1.14	140	3.8	518.5	483
	-6.	0.11	214	1.02	0.76	0.77	1.35	141	1.4	499.6	111

Table 3.2: Ice drift Fram Strait January 1994 starting in circles of radius 25 km and centers at N 79.0°.

Start		3 days mean		Deformation					Rota-	Prediction	#
date (1994)	longi- tude	Vel (m/s)	Dir (deg)	v_1	v_2	$v_1 v_2$	v_1/v_2	θ_1 (deg)	tion (deg)	error (m)	data points
01.01	-10.	0.28	203	1.24	0.81	1.00	1.52	120	13.5	3659.0	526
	-9.	0.32	200	1.37	0.81	1.10	1.70	133	17.4	3331.9	1025
	-8.	0.34	200	1.09	0.97	1.05	1.12	138	5.9	1249.1	1339
	-7.	0.35	200	1.15	0.88	1.01	1.30	133	10.7	1898.6	713
01.04	-10.	0.29	209	1.07	0.95	1.01	1.12	111	7.8	1419.3	748
	-9.	0.30	208	1.13	0.94	1.07	1.20	120	6.9	1543.5	1222
	-8.	0.31	206	1.16	0.91	1.05	1.28	123	3.9	1353.9	1009
	-7.	0.32	206	1.20	0.86	1.03	1.39	131	2.3	1250.5	565
01.07	-10.	0.24	199	1.09	0.90	0.98	1.21	115	0.0	951.7	557
	-9.	0.24	199	1.06	0.94	1.00	1.13	108	-0.7	591.0	1166
	-8.	0.24	199	1.04	0.96	1.00	1.09	111	0.6	570.0	1247
	-7.	0.24	199	1.03	0.98	1.01	1.06	115	1.5	417.8	1107
01.10	-10.	0.17	198	1.05	0.93	0.99	1.13	140	7.4	755.9	305
	-9.	0.18	197	1.13	0.90	1.02	1.25	124	7.7	1136.9	741
	-8.	0.20	195	1.15	0.90	1.04	1.28	113	6.2	1146.8	898
	-7.	0.21	193	1.23	0.91	1.12	1.35	116	4.6	1169.7	664
01.13	-10.	0.29	198	1.46	0.75	1.10	1.96	132	24.6	2595.4	288
	-9.	0.32	197	1.26	0.77	0.97	1.62	127	13.8	3099.7	734
	-8.	0.34	196	1.16	0.85	0.99	1.36	116	7.2	1192.5	954
	-7.	0.35	195	1.12	0.87	0.97	1.29	118	7.8	729.1	552
01.16	-10.	0.15	197	1.16	0.93	1.08	1.26	115	2.3	1447.5	627
	-9.	0.17	196	1.15	0.89	1.02	1.29	139	8.2	1418.0	1089
	-8.	0.18	196	1.05	0.96	1.00	1.09	138	4.0	1030.0	1314
	-7.	0.19	197	1.02	0.98	1.00	1.04	128	2.7	347.3	1310
01.19	-10.	0.27	217	1.07	0.85	0.91	1.26	25	4.2	547.1	256
	-9.	0.27	219	1.05	0.87	0.91	1.20	28	1.1	653.8	619
	-8.	0.27	221	1.01	0.93	0.94	1.08	28	2.0	498.2	903
	-7.	0.27	222	0.98	0.97	0.95	1.02	80	3.2	343.2	1047
01.22	-10.	0.35	207	1.27	0.87	1.10	1.47	133	12.0	1393.8	479
	-9.	0.37	206	1.35	0.84	1.14	1.61	133	15.0	1530.4	668
	-8.	0.41	205	1.42	0.83	1.19	1.71	138	13.3	1526.0	546
	-7.	0.43	204	1.42	0.83	1.18	1.71	144	11.3	1618.7	303
01.31	-10.	0.09	215	1.01	0.96	0.97	1.05	170	3.8	199.4	495
	-9.	0.10	215	1.00	0.97	0.97	1.03	2	4.1	213.5	1134
	-8.	0.10	216	1.00	0.97	0.96	1.03	179	4.5	223.1	1269
	-7.	0.11	216	1.00	0.96	0.96	1.04	138	5.2	406.6	962

Table 3.3: Ice drift Fram Strait January 1994 starting in circles of radius 25 km and centers at N 78.7°.

Start		3 days mean		Deformation					Rota-	Prediction	#
date	longi-	Vel	Dir					θ_1	tion	error	data
(1994)	tude	(m/s)	(deg)	v_1	v_2	$v_1 v_2$	v_1/v_2	(deg)	(deg)	(m)	points
01.01	-11.	0.25	205	1.13	0.84	0.95	1.35	93	-2.3	1330.6	502
	-10.	0.27	204	1.33	0.75	1.00	1.77	122	15.4	5093.7	838
	-9.	0.32	200	1.37	0.70	0.95	1.96	127	18.7	4640.7	960
	-8.	0.34	199	1.12	0.96	1.07	1.16	128	0.7	1264.0	968
01.04	-11.	0.28	210	1.42	0.91	1.30	1.56	101	0.5	1650.2	240
	-10.	0.29	208	1.32	0.88	1.15	1.50	121	5.3	2383.4	831
	-9.	0.29	207	1.21	0.89	1.08	1.36	128	4.4	2170.4	1098
	-8.	0.31	205	1.16	0.92	1.06	1.26	126	3.8	1126.7	964
01.07	-11.	0.20	203	1.23	0.74	0.91	1.67	119	17.2	2710.3	420
	-10.	0.23	200	1.14	0.83	0.95	1.38	121	11.5	2859.6	933
	-9.	0.24	200	1.01	0.98	1.00	1.03	93	2.5	235.0	1374
	-8.	0.24	200	1.03	0.98	1.01	1.06	96	2.5	448.8	1190
01.10	-11.	0.14	182	1.07	1.04	1.11	1.03	119	9.2	1223.7	195
	-10.	0.15	181	1.12	0.90	1.01	1.24	114	2.1	2218.2	722
	-9.	0.16	182	1.10	0.84	0.93	1.32	131	1.5	2818.5	996
	-8.	0.17	183	1.08	0.73	0.79	1.49	141	1.0	2783.8	592
01.13	-11.	0.24	201	1.32	0.81	1.07	1.63	134	15.8	1733.0	344
	-10.	0.28	200	1.34	0.81	1.08	1.66	129	20.0	1710.9	632
	-9.	0.31	198	1.14	0.84	0.96	1.35	122	13.8	1653.5	908
	-8.	0.33	197	1.10	0.90	0.99	1.22	107	8.4	856.2	827
01.16	-11.	0.13	197	1.08	0.92	1.00	1.17	111	2.4	1298.3	751
	-10.	0.15	197	1.14	0.87	0.99	1.31	142	9.7	2260.7	1082
	-9.	0.17	197	1.10	0.89	0.98	1.24	147	9.5	1999.6	944
	-8.	0.18	198	1.04	0.97	1.01	1.07	124	3.6	418.9	1036
01.19	-11.	0.27	217	1.01	0.92	0.93	1.09	23	0.3	252.6	268
	-10.	0.27	218	1.01	0.92	0.93	1.09	42	0.0	365.8	757
	-9.	0.27	220	1.01	0.93	0.94	1.09	44	-0.6	383.3	951
	-8.	0.27	222	1.00	0.96	0.96	1.04	42	0.1	345.0	997
01.22	-11.	0.34	208	1.08	0.93	1.00	1.16	139	6.0	449.8	367
	-10.	0.35	208	1.22	0.84	1.02	1.45	140	12.6	2271.0	664
	-9.	0.40	206	1.31	0.87	1.14	1.51	136	20.0	1872.8	668
	-8.	0.42	205	1.23	0.94	1.16	1.32	136	18.8	1397.8	527
01.31	-11.	0.09	216	1.00	0.93	0.93	1.08	12	3.0	468.0	185
	-10.	0.09	217	1.01	0.94	0.95	1.08	5	2.2	341.9	185
	-9.	0.10	219	1.00	0.97	0.96	1.03	171	2.0	189.0	219
	-8.	0.10	219	0.99	0.95	0.93	1.04	109	1.5	323.2	425

Table 3.4: Ice drift Fram Strait February 1994 starting in circles of radius 25 km and centers at N 79.3°.

Start		3 days mean		Deformation					Rota-	Prediction	#
date	longi-	Vel	Dir					θ_1	tion	error	data
(1994)	tude	(m/s)	(deg)	v_1	v_2	$v_1 v_2$	v_1/v_2	(deg)	(deg)	(m)	points
02.03	-9.	0.09	193	1.08	0.99	1.07	1.09	128	4.4	456.1	621
	-8.	0.09	192	1.04	0.99	1.03	1.06	113	3.4	529.0	1008
	-7.	0.09	192	1.02	0.99	1.01	1.03	73	1.5	284.7	760
	-6.	0.09	193	1.03	0.96	0.99	1.08	40	2.9	819.8	229
02.06	-9.	0.18	217	1.03	0.96	0.99	1.08	179	-0.7	356.4	642
	-8.	0.19	218	1.03	0.97	1.01	1.06	172	0.6	321.5	883
	-7.	0.19	218	1.03	0.97	1.00	1.06	8	0.7	329.3	548
	-6.	0.19	220	1.05	0.96	1.00	1.09	30	-0.1	283.5	126
02.09	-9.	0.03	23	1.03	0.93	0.95	1.11	85	1.4	712.0	91
	-8.	0.03	27	0.98	0.91	0.90	1.08	98	0.7	624.9	34
02.12	-9.	0.03	39	1.05	0.94	0.98	1.12	61	-8.0	1058.8	54
	-8.	0.03	34	1.12	0.90	1.01	1.24	72	-6.1	933.0	32
02.15	-9.	0.13	213	1.08	0.92	0.99	1.18	170	10.2	1186.5	781
	-8.	0.13	213	1.07	0.92	0.98	1.16	168	9.6	1055.5	824
	-7.	0.14	213	1.06	0.92	0.97	1.14	3	9.8	861.8	275
	-6.	0.16	216	1.15	0.81	0.93	1.41	35	7.9	664.0	27
02.18	-9.	0.23	214	1.03	0.99	1.02	1.05	164	6.6	482.7	559
	-8.	0.24	214	1.03	0.99	1.01	1.04	171	5.4	351.7	812
	-7.	0.24	214	1.01	1.00	1.01	1.02	2	4.8	138.1	452
	-6.	0.24	214	1.01	0.99	1.00	1.02	12	4.8	68.8	71
02.21	-9.	0.27	203	1.28	0.86	1.09	1.49	146	15.1	1663.9	492
	-8.	0.29	202	1.15	0.93	1.07	1.24	144	12.1	1524.8	979
	-7.	0.31	203	1.01	0.98	0.98	1.03	134	8.3	310.0	834
	-6.	0.31	203	1.01	0.98	0.99	1.04	123	7.7	115.8	416
02.24	-9.	0.06	187	1.03	0.97	1.00	1.06	151	-1.1	1177.8	699
	-8.	0.06	184	1.11	0.97	1.08	1.14	103	-1.7	1159.7	903
	-7.	0.06	179	1.10	0.97	1.06	1.14	106	-5.0	1062.3	423
	-6.	0.06	172	1.60	0.29	0.46	5.53	90	30.7	1678.3	10
02.27	-9.	0.11	214	1.05	0.91	0.95	1.15	175	9.7	918.9	726
	-8.	0.11	214	1.01	0.93	0.94	1.09	14	9.0	1196.5	853
	-7.	0.12	216	1.03	0.90	0.93	1.15	53	8.4	821.3	643
	-6.	0.13	218	1.00	0.99	1.00	1.01	161	7.6	71.2	360

Table 3.5: Ice drift Fram Strait February 1994 starting in circles of radius 25 km and centers at N 79.0°.

Start		3 days mean		Deformation					Rota-	Prediction	#
date	longi-	Vel	Dir					θ_1	tion	error	data
(1994)	tude	(m/s)	(deg)	v_1	v_2	$v_1 v_2$	v_1/v_2	(deg)	(deg)	(m)	points
02.03	-10.	0.09	193	1.07	0.98	1.05	1.09	143	1.8	465.5	668
	-9.	0.09	193	1.06	0.98	1.04	1.09	140	1.1	465.7	1127
	-8.	0.09	192	1.03	0.98	1.00	1.05	131	0.6	446.0	1024
	-7.	0.09	193	1.04	0.95	0.99	1.10	146	1.2	886.8	714
02.06	-10.	0.18	213	1.08	0.95	1.03	1.15	151	1.3	743.8	629
	-9.	0.19	214	1.09	0.96	1.04	1.13	160	-0.1	652.0	1170
	-8.	0.19	215	1.07	0.97	1.04	1.10	168	-2.6	628.9	1289
	-7.	0.19	217	1.03	0.98	1.01	1.05	158	-4.4	461.6	1038
02.09	-10.	0.03	358	1.00	0.90	0.90	1.11	46	5.2	604.8	111
	-9.	0.03	7	1.10	0.93	1.03	1.18	79	3.7	852.9	160
	-8.	0.03	18	1.16	0.90	1.04	1.30	63	-3.3	1064.2	40
	-7.	0.06	19	1.13	0.69	0.78	1.63	144	-17.6	15.1	3
02.12	-10.	0.03	65	1.17	1.01	1.18	1.16	122	-3.1	1440.3	32
	-9.	0.03	57	1.04	1.01	1.05	1.03	80	-8.7	1308.9	46
02.15	-10.	0.13	214	1.15	0.81	0.93	1.41	156	6.6	1264.0	484
	-9.	0.14	214	1.14	0.88	1.00	1.29	157	5.7	1297.8	995
	-8.	0.14	214	1.13	0.89	1.01	1.27	156	3.7	1084.4	800
	-7.	0.16	216	1.10	0.89	0.98	1.23	174	-0.5	1416.4	296
02.18	-10.	0.23	213	1.25	0.96	1.20	1.30	154	-1.2	1680.2	530
	-9.	0.24	213	1.19	0.94	1.12	1.26	149	1.9	1363.7	952
	-8.	0.24	213	1.13	0.94	1.06	1.21	157	2.1	969.8	838
	-7.	0.25	213	1.09	0.96	1.04	1.13	168	0.8	818.7	454
02.21	-10.	0.27	206	1.15	0.89	1.02	1.29	143	12.2	1322.0	618
	-9.	0.28	205	1.13	0.93	1.05	1.22	144	10.9	1444.3	983
	-8.	0.30	204	1.06	0.96	1.01	1.11	141	8.6	721.5	988
	-7.	0.31	204	1.03	0.98	1.01	1.05	134	7.7	339.1	788
02.24	-10.	0.06	169	1.06	0.97	1.03	1.10	162	-4.1	1018.1	624
	-9.	0.06	171	1.03	0.98	1.00	1.05	125	-7.7	1047.1	1314
	-8.	0.05	169	1.08	0.93	1.00	1.17	109	-5.0	1067.6	1592
	-7.	0.06	163	1.23	0.90	1.11	1.37	120	1.1	1650.9	1198
02.27	-10.	0.11	212	1.14	0.86	0.98	1.33	168	2.6	948.8	550
	-9.	0.11	215	1.09	0.90	0.98	1.21	172	0.7	1111.7	1028
	-8.	0.12	218	1.01	0.95	0.96	1.07	172	-2.3	903.2	1329
	-7.	0.12	220	1.02	0.89	0.91	1.15	129	1.2	1361.2	1170

Table 3.6: Ice drift Fram Strait February 1994 starting in circles of radius 25 km and centers at N 78.7°.

Start		3 days mean		Deformation					Rota-	Prediction	#
date (1994)	longi- tude	Vel (m/s)	Dir (deg)	v_1	v_2	$v_1 v_2$	v_1/v_2	θ_1 (deg)	tion (deg)	error (m)	data points
02.03	-11.	0.08	190	1.20	0.96	1.15	1.25	132	3.1	595.9	169
	-10.	0.09	191	1.10	0.97	1.07	1.13	142	0.0	669.8	571
	-9.	0.09	191	1.05	0.97	1.02	1.07	129	-1.4	470.2	664
	-8.	0.09	190	1.07	0.96	1.02	1.11	117	-1.1	661.8	567
02.06	-11.	0.18	211	1.08	0.98	1.06	1.11	146	3.6	960.6	624
	-10.	0.19	211	1.07	0.99	1.07	1.08	146	1.5	750.8	1295
	-9.	0.19	212	1.03	0.98	1.01	1.05	137	-0.4	494.3	1382
	-8.	0.19	213	1.01	0.98	1.00	1.03	49	-3.5	538.2	1155
02.09	-11.	0.04	354	1.01	0.97	0.97	1.04	84	-2.5	653.7	217
	-10.	0.04	354	1.03	0.95	0.97	1.08	87	-3.5	668.9	290
	-9.	0.04	357	1.11	0.91	1.01	1.21	98	-2.4	655.3	116
	-8.	0.04	6	1.12	0.78	0.87	1.43	79	-0.4	424.3	20
02.12	-11.	0.04	54	1.21	0.92	1.11	1.31	136	-10.5	818.5	111
	-10.	0.03	42	1.30	0.91	1.18	1.42	101	2.6	1186.4	68
	-9.	0.04	71	1.30	0.85	1.11	1.53	117	-16.1	278.3	8
02.15	-11.	0.12	213	1.11	0.92	1.03	1.21	160	11.2	951.3	606
	-10.	0.13	212	1.11	0.92	1.03	1.21	156	9.1	1441.9	1059
	-9.	0.15	212	1.14	0.86	0.98	1.33	170	10.1	2155.3	1055
	-8.	0.16	213	1.12	0.87	0.98	1.29	1	9.0	1996.3	651
02.18	-11.	0.24	208	1.05	0.95	1.01	1.10	150	3.9	544.1	554
	-10.	0.24	208	1.07	0.95	1.02	1.13	161	2.5	821.6	862
	-9.	0.24	209	1.10	0.91	1.00	1.21	167	1.9	769.0	697
	-8.	0.25	211	1.07	0.94	1.01	1.14	165	-0.1	838.0	452
02.21	-11.	0.25	201	1.40	0.68	0.95	2.07	159	-0.5	3744.2	444
	-10.	0.27	204	1.35	0.72	0.97	1.87	159	0.9	3695.3	905
	-9.	0.28	205	1.33	0.86	1.14	1.55	144	4.2	3129.0	786
	-8.	0.30	206	1.02	0.95	0.97	1.08	157	9.2	725.4	326
02.24	-11.	0.07	161	1.12	0.93	1.04	1.20	114	-1.1	868.5	526
	-10.	0.07	160	1.07	0.96	1.03	1.12	89	-6.1	853.9	1165
	-9.	0.06	156	1.02	0.98	1.00	1.05	77	-7.2	408.9	1588
	-8.	0.05	153	1.03	0.98	1.01	1.05	116	-4.7	791.9	1535
02.27	-11.	0.10	204	1.17	0.68	0.80	1.72	160	1.8	2072.4	415
	-10.	0.11	208	1.09	0.91	0.99	1.19	162	0.6	1280.8	985
	-9.	0.12	211	1.01	0.97	0.98	1.04	176	-3.4	388.4	1399
	-8.	0.11	214	1.00	0.99	0.99	1.02	144	-4.7	277.2	1450

Table 3.7: Ice drift Fram Strait March 1994 starting in circles of radius 25 km and centers at N 79.3°.

Start		3 days mean		Deformation					Rota-	Prediction	#
date	longi-	Vel	Dir					θ_1	tion	error	data
(1994)	tude	(m/s)	(deg)	v_1	v_2	$v_1 v_2$	v_1/v_2	(deg)	(deg)	(m)	points
03.02	-9.	0.33	215	1.07	0.94	1.01	1.13	169	2.0	497.1	576
	-8.	0.34	215	1.07	0.94	1.01	1.14	170	2.5	546.9	834
	-7.	0.34	216	1.05	0.95	1.00	1.10	170	1.3	598.2	634
	-6.	0.34	217	1.00	0.99	0.99	1.01	108	-1.6	179.2	286
03.05	-9.	0.35	207	1.08	0.93	1.00	1.17	147	8.3	673.1	651
	-8.	0.36	207	1.07	0.93	1.00	1.14	141	8.4	587.5	838
	-7.	0.37	207	1.05	0.93	0.98	1.14	131	9.4	634.6	426
	-6.	0.38	207	1.06	0.92	0.97	1.14	116	10.4	570.6	64
03.08	-9.	0.15	196	1.02	0.99	1.00	1.03	168	6.6	179.9	870
	-8.	0.16	197	1.02	0.98	1.00	1.04	162	6.6	161.1	1100
	-7.	0.17	197	1.02	0.97	0.99	1.04	148	6.7	200.8	664
	-6.	0.17	198	1.03	0.94	0.97	1.09	137	7.3	237.1	141
03.20	-9.	0.14	187	1.05	0.96	1.01	1.10	96	9.3	374.3	939
	-8.	0.15	186	1.05	0.96	1.01	1.09	96	9.8	336.3	1300
	-7.	0.16	187	1.04	0.98	1.01	1.06	96	10.6	334.5	880
	-6.	0.17	188	1.04	0.99	1.03	1.05	67	11.0	430.8	373
03.23	-9.	0.18	205	1.01	0.97	0.98	1.03	61	4.9	204.5	953
	-8.	0.18	205	1.04	0.97	1.00	1.07	64	4.0	395.1	1101
	-7.	0.18	205	1.09	0.95	1.03	1.14	72	4.1	542.9	596
	-6.	0.18	205	1.11	0.99	1.10	1.11	76	1.2	444.4	241
03.26	-9.	0.27	197	1.14	0.86	0.98	1.33	102	0.2	1251.9	814
	-8.	0.28	197	1.09	0.89	0.97	1.22	105	3.2	1349.7	1216
	-7.	0.28	197	1.04	0.89	0.93	1.16	122	8.3	1339.6	806
	-6.	0.29	197	1.12	0.77	0.86	1.45	124	16.7	1367.8	285

Table 3.8: Ice drift Fram Strait March 1994 starting in circles of radius 25 km and centers at N 79.0°.

Start date (1994)	Start longitude	3 days mean Vel (m/s)	3 days mean Dir (deg)	Deformation v_1	Deformation v_2	Deformation $v_1 v_2$	Deformation v_1/v_2	Deformation θ_1 (deg)	Rotation (deg)	Prediction error (m)	# data points
03.02	-10.	0.33	214	1.04	0.92	0.96	1.13	6	3.7	665.7	541
	-9.	0.33	214	1.01	0.94	0.95	1.08	175	2.7	714.2	1147
	-8.	0.34	215	1.01	0.96	0.97	1.06	155	2.0	640.3	1467
	-7.	0.34	216	1.00	1.00	1.00	1.00	100	-1.0	132.2	1494
03.05	-10.	0.34	208	1.06	0.97	1.03	1.10	156	8.0	959.1	705
	-9.	0.35	208	1.02	0.97	0.99	1.05	135	6.9	759.4	1307
	-8.	0.36	208	1.04	0.93	0.97	1.11	106	5.1	878.3	1239
	-7.	0.37	208	1.08	0.88	0.95	1.23	122	8.3	1361.0	690
03.08	-10.	0.15	199	1.08	1.00	1.08	1.08	165	3.6	472.8	763
	-9.	0.16	199	1.06	0.99	1.05	1.06	156	3.8	513.7	1391
	-8.	0.16	199	1.05	0.97	1.02	1.08	139	3.5	490.1	1493
	-7.	0.17	198	1.10	0.94	1.04	1.16	126	4.2	1260.3	1116
03.20	-10.	0.13	198	1.08	0.98	1.06	1.11	42	9.8	634.0	944
	-9.	0.13	195	1.05	0.99	1.04	1.05	47	10.7	554.1	1487
	-8.	0.15	194	1.02	1.00	1.02	1.03	64	11.9	442.2	1520
	-7.	0.16	194	1.02	1.00	1.02	1.03	72	11.6	427.4	1534
03.23	-10.	0.18	209	1.01	0.99	1.00	1.02	81	6.3	268.6	967
	-9.	0.18	208	1.03	0.98	1.01	1.05	79	5.0	374.7	1565
	-8.	0.18	208	1.04	0.99	1.02	1.05	74	2.6	537.7	1395
	-7.	0.18	207	1.05	0.97	1.02	1.09	71	2.0	712.3	1243
03.26	-10.	0.26	199	1.02	0.96	0.98	1.06	114	7.7	464.4	967
	-9.	0.26	199	1.01	0.97	0.98	1.04	120	7.9	633.7	1555
	-8.	0.28	199	1.00	0.99	1.00	1.01	153	8.4	211.9	1652
	-7.	0.29	199	1.03	0.97	1.00	1.06	139	9.8	922.8	1251

Table 3.9: Ice drift Fram Strait March 1994 starting in circles of radius 25 km and centers at N 78.7°.

Start		3 days mean		Deformation					Rota-	Prediction	#
date (1994)	longi- tude	Vel (m/s)	Dir (deg)	v_1	v_2	$v_1 v_2$	v_1/v_2	θ_1 (deg)	tion (deg)	error (m)	data points
03.02	-11.	0.33	211	1.13	0.72	0.82	1.55	172	0.9	1676.6	322
	-10.	0.34	213	1.11	0.83	0.93	1.34	177	1.9	1539.3	942
	-9.	0.34	214	1.09	0.97	1.07	1.12	10	0.8	796.2	1311
	-8.	0.34	215	1.03	1.01	1.04	1.02	10	0.4	903.7	1204
03.05	-11.	0.35	206	1.11	0.92	1.02	1.21	177	-1.6	924.4	375
	-10.	0.35	207	1.11	0.94	1.04	1.17	165	-0.3	1055.7	972
	-9.	0.35	208	1.04	0.97	1.01	1.07	154	0.0	669.5	1325
	-8.	0.36	209	1.02	0.96	0.98	1.07	128	0.7	713.9	1023
03.08	-11.	0.15	200	1.02	0.99	1.00	1.03	133	2.4	273.1	787
	-10.	0.16	200	1.01	0.99	1.00	1.02	135	2.6	227.0	1500
	-9.	0.16	200	1.01	0.99	1.00	1.02	133	3.2	230.3	1613
	-8.	0.16	200	1.02	0.99	1.01	1.03	136	4.6	905.2	1337
03.20	-11.	0.13	207	1.04	0.98	1.02	1.06	33	4.2	490.5	905
	-10.	0.13	206	1.03	0.99	1.02	1.05	51	5.6	615.6	1423
	-9.	0.14	205	1.03	0.97	1.00	1.06	96	7.9	718.9	1413
	-8.	0.15	204	1.03	0.99	1.02	1.05	120	9.6	736.4	1491
03.23	-11.	0.18	214	1.04	0.97	1.01	1.06	18	4.6	697.4	708
	-10.	0.18	213	1.04	1.00	1.05	1.04	80	4.2	916.0	1313
	-9.	0.18	212	1.06	0.98	1.04	1.08	93	3.7	761.2	1349
	-8.	0.18	211	1.08	0.97	1.05	1.12	84	2.5	661.1	1256
03.26	-11.	0.25	202	1.01	0.99	0.99	1.02	154	6.4	174.2	974
	-10.	0.25	202	1.02	0.98	1.00	1.04	155	7.6	262.6	1675
	-9.	0.27	202	1.02	0.98	1.00	1.05	145	8.4	348.0	1406
	-8.	0.28	202	1.04	0.96	1.00	1.09	120	5.8	861.2	1230

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Appendix A

Representation of ice drift

A set of ice drift vectors can be expressed in a Taylor-like expansion:

$$\boldsymbol{T}_{t_1,t_2}(\mathbf{x}) = \bar{\mathbf{y}} + \mathbf{A}(\mathbf{x} - \bar{\mathbf{x}}) + \boldsymbol{\epsilon} \tag{A.1}$$

The symbols here are as in (Korsnes, 1994a). $\boldsymbol{T}_{t_1,t_2}(\mathbf{x})$ gives the position at time t_2 of the ice point which at the time t_1 is at $\mathbf{x}$. $\mathbf{A}$ is a 2×2 matrix. $\boldsymbol{\epsilon}$ is an error or model failure term. The vector $\mathbf{b} = \bar{\mathbf{y}} - \bar{\mathbf{x}}$ reflects ice drift. The transformation $\mathbf{A}$ maps a unit circle into an ellipsis with axes of length $v_i(\mathbf{A}) = \{\lambda_i(\mathbf{A}^T\mathbf{A})\}^{1/2}$, $i = 1, 2$, where $\lambda_i(\mathbf{A}^T\mathbf{A})$ are the eigenvalues with the corresponding eigenvectors $\mathbf{e}_i$ of the symmetric, positive semidefinite, matrix $\mathbf{A}^T\mathbf{A}$. We assume $\lambda_1 \geq \lambda_2$. $v_i(\mathbf{A})$ are in the literature called *singular values* of the matrix $\mathbf{A}$ (Ciarlet, 1988, Chapter 3). We can interpret $v_1(\mathbf{A})v_2(\mathbf{A})$ as the increase of area between the image sample times. Let

$$\mathbf{A} = \mathbf{CU} \tag{A.2}$$

be the (unique) polar factorization of (the invertible) matrix $\mathbf{A}$ where $\mathbf{C}$ is a rotation and $\mathbf{U}$ is a positive definite matrix (note: we can assume determinant($\mathbf{A}$) > 0). $\mathbf{U}$ reflects deformation given by its eigenvalues v_i, $i = 1, 2$, and direction θ_1 of the eigenvector $\mathbf{e}_1$.